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Information causality: An operational principle in limiting nonlocality in nature

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Abstract: Quantum mechanics exhibits non-locality while simultaneously adhering to the no-signaling condition, which ensures that information cannot be transmitted faster than the speed of light. However, the extent of non-local correlations in quantum mechanics is fundamentally limited.

A natural question arises: Does the no-signaling condition itself impose a strict upper bound on the degree of non-local correlations? Interestingly, the answer is no. The no-signaling condition permits correlations that are even stronger than those observed in quantum mechanics, such as those found in the Popescu-Rohrlich (PR) box framework.

Surprisingly, a recently proposed principle known as **information causality** provides a deeper constraint, ensuring that the strength of non-local correlations remains within the quantum limit. This principle extends beyond the no-signaling condition and plays a crucial role in distinguishing quantum mechanics from more generalized non-signaling theories.

Keywords: No-Signaling; Popescu Rohrlich Box; Information Causality

1. Introduction

In the pre-Einsteinian era, the concept of superluminal communication was an accepted part of theoretical physics. However, it was Einstein who rigorously eliminated it by formulating a framework that upheld locality, leading to the development of the rich theories of special relativity and, later, general relativity.

Ironically, with the advent of quantum mechanics, Einstein himself became increasingly troubled by the emergence of non-local features within the theory. Despite numerous efforts, he was unable to pinpoint a definitive, uncontested instance of non-locality within quantum formalism.

In 1935, building on the principles of realism and locality, Einstein, along with his colleagues Boris Podolsky and Nathan Rosen, argued that although quantum mechanics was internally consistent, it remained incomplete. This conclusion, now famously known as the **EPR paradox**, laid the foundation for future debates on the nature of quantum entanglement and non-local correlations [1].

In 1964, J. S. Bell made a groundbreaking discovery by identifying the non-local features of quantum theory in a precise and meaningful way. His work revealed that quantum mechanics exhibits correlations that cannot be explained by any local hidden variable theory, as formalized in **Bell's theorem**.

With Bell's discovery, it became evident that a non-local theory does not necessarily contradict the principles of relativity—specifically, the **no-signaling condition**, which prohibits the instantaneous transfer of information. Quantum mechanics, despite its non-local correlations, does not enable faster-than-light communication, thereby remaining consistent with special relativity. However, this raises a deeper question: In what way is quantum mechanics non-local? Is its non-locality akin to that of Newton's theory of gravitation, which postulates instantaneous action at a distance? The distinction between quantum non-locality and classical non-locality remains a subject of fundamental inquiry, highlighting the subtle and counterintuitive nature of entanglement and quantum correlations.

If we assume any theory that respects the following two conditions:

- Measurement of some observable on a system only reveals the pre-existing values (realism),
- Result of measurement performed on one system in one location does not depend on which measurement is performed on another system in different location, in whichever way correlated they may be (locality),

Then, any such theory cannot reproduce all the results of quantum mechanics and quantum mechanics is considered to be non-local in this sense only.

2. Two party Toy game

To understand this question more clearly, we present a game played between two players located in different positions. Let Alice and Bob are kept in two different rooms. Both of them will be given a question chosen from the pair$(a_i, b_j;\ i, j = 1,2)$, where the first one from the pair goes to Alice and second one goes to Bob. The pair will be chosen randomly from the above set. The answer can be +1 or -1. After the game starts Alice and Bob will not be allowed to communicate. But before the game starts, they can meet and design any strategy and also share correlated physical systems. The game will be repeated many times and in the end their answers will be collected to verify whether those answers satisfy the following winning conditions in all the cases:

For the pair (a_1, b_1), the product of their answers should be +1

For the pair (a_1, b_2), the product of their answers should be +1

For the pair (a_2, b_1), the product of their answers should be +1

For the pair (a_2, b_2), the product of their answers should be -1

So, the winning condition has nothing to do with their individual answer, only product of their answers has to satisfy the above conditions.

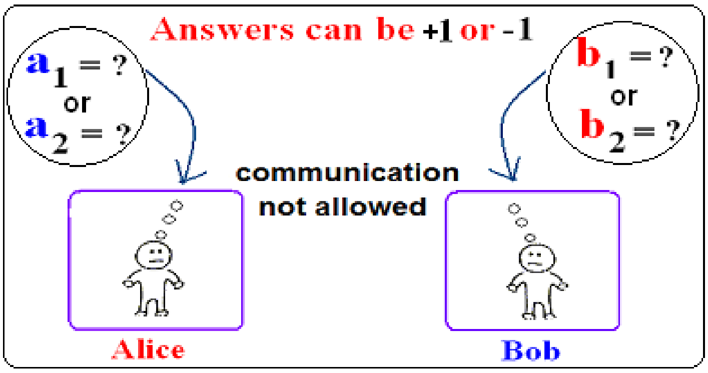

Fig.1

Let there is a strategy by which Alice and Bob can win this game in the classical world. Then in each turn, after exploiting all the correlations shared among them through physical systems they possess, the answers to be given by Alice and Bob are fixed locally. Let in particular turn, $v_{Alice}(a_1)$ and $v_{Alice}(a_2)$ are Alice's answer for questions a_1 *and* a_2 respectively. Similarly, $v_{Bob}(b_1)$ and $v_{Bob}(b_2)$ are Bob's answer for questions b_1 *and* b_2 respectively. Now for winning the game these answers have to satisfy all the four conditions as all pairs of questions are equally likely and they have to be correct in all cases. Then the answers have to satisfy the following conditions:

$$v_{Alice}(a_1)v_{Bob}(b_1) = 1$$
$$v_{Alice}(a_1)v_{Bob}(b_2) = 1$$
$$v_{Alice}(a_2)v_{Bob}(b_1) = 1$$
$$v_{Alice}(a_2)v_{Bob}(b_2) = -1$$

If we take the product of the four equations, the left-hand side will be positive as every term has appeared twice and answers can be either +1 or -1, whereas the right-hand side will be -1. This shows that it is impossible to win this game not only in classical world but it is even true for any hypothetical theory which respects locality and realism.

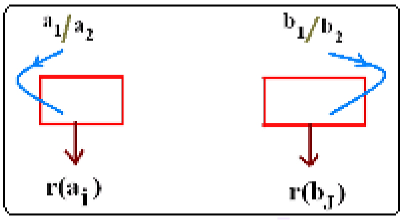

Fig.2

3. Popescu Rohrlich Box

In this context let us now imagine the existence of a pair of correlated magic boxes shared by two parties who can be far away from each other. There are two possible inputs to both the boxes and one input can be given at a time. Let a_1 and a_2 denote inputs for the box on the left-hand side and

b_1, b_2 denote input for the box on the right-hand side (Figure 2). We assume that the inputs to the boxes and corresponding outputs, $r(a_i)(\, i = 1,2)$ for the first box and $r(b_j)(\, j = 1,2)$ for the second box, are correlated in a way that can help win the game and this correlation does not depend on the distance between the boxes. Let the correlation is given by either of the following two shown in Figure 3a and Figure 3b.

Input		Output	
a_1	b_1	+1	+1
a_1	b_2	+1	+1
a_2	b_1	+1	+1
a_2	b_2	+1	-1

Fig.3a

Input		Output	
a_1	b_1	-1	-1
a_1	b_2	-1	-1
a_2	b_1	-1	-1
a_2	b_2	-1	+1

Fig.3b

These correlations have the features that for box on the left, the output is deterministically determined. But for the box on the right, though the output is determined for input b_1, it is not so for b_2, output in this case depends on input to the other box. Hence these correlations do not respect no-signaling condition. Let us see how. Let Aloke and Binoy each share one box from the pair of magic boxes and their inputs and outputs obey the correlation shown in Figure 3a. Let they are light year away and has agreed a fixed time in future when Aloke has to inform the result of a cricket match to be held between India and Pakistan. We can see that the result can be immediately sent with the help of the magic box. If Indian wins Aloke inputs a_1 to his box and in case India loses he inputs a_2. Binoy always inputs b_2. Now, from the correlation shown in fig.3a, one can easily see that if the output for Binoy is +1, he will understand that Aloke's input is a_1 and hence following their strategy, he would know that India wins. On the other hand, if the output for Binoy is -1, he will learn that Aloke's input is a_2 and he would know that India loses. Similar thing will happen for the correlation shown in Figure 3b.

So, any correlation for (non-local) deterministic output that could help to win the game will violate the postulate of special relativity. Now the question arises whether there is any correlation that could help to win the game still respects no-signaling condition. The answer may be affirmative if there is a proper marriage between non local correlation and non-deterministic output in the box *i.e.* a proper mixing between non-local correlation and probability has to be introduced. Keeping this in mind, two scientists Popescu and Rohrlich [3] suggested a new correlation in 1996 (Figure 4) between input and output for the pair of magic boxes where the output for each input is completely random. For example, when the inputs are a_1 and b_1, the corresponding outputs are strictly correlated (either both +1 or both -1) but both the results are equally probable.

Input		Output		Probability
a_1	b_1	+1	+1	$\frac{1}{2}$
		-1	-1	$\frac{1}{2}$
a_1	b_2	+1	+1	$\frac{1}{2}$
		-1	-1	$\frac{1}{2}$
a_2	b_1	+1	+1	$\frac{1}{2}$
		-1	-1	$\frac{1}{2}$
a_2	b_2	-1	+1	$\frac{1}{2}$
		+1	-1	$\frac{1}{2}$

Fig.4

In this case, one can easily check that all the results on either side are completely random and hence from the output of any box, input to the other cannot be determined. Hence it respects the no-signaling condition. But this correlation is still helpful for Alice and Bob to win the game suggested in the beginning. So, this is a correlation which respects no-signaling condition but at the same time cannot be generated by any local realistic theory and hence no classical correlation or shared randomness can be used to generate such correlation.

Now the magic box correlation being consistent with special relativity, the interesting question that arises is; Does nature provides the correlation exhibited by the magic boxes? The strict answer to this question is No. But the issue is more involved. In quantum world, there are some correlations produced by measurement on correlated quantum system, which also cannot be reproduced by classical or any local realistic theory. How this is understood? It is shown that if the pair of questions that are given to Alice and Bob is random, then in classical world or for any theory which is local realistic, the probability of winning the game can be at most $\frac{3}{4}$ i.e. they can win in 75% cases. The simple strategy will be Alice and Bob will always put their answer to be +1. Then they will only lose in one of the four cases when the pair of question is a_2 and b_2. But interestingly, there is a quantum strategy by which they can win the game in 85% cases. For quantum world, the strategy is as follows;

Alice and Bob share many copies of a quantum state of two spin-1/2 systems. The quantum state is singlet and it has the nice property that if Alice and Bob measure spin along same direction on their respective particles, their measurement results are strictly anti-correlated. They use each singlet state for answering each pair of questions. If Alice is asked question a_1 she measures spin on her particle along $\hat{a}_1$ direction (Z-axis in Figure 5) and along $\hat{a}_2$ direction for question a_2 (along x axis in the fig.5). Similarly for question b_1 Bob measures along direction $\hat{b}_1$ which makes 225^0 with x-axis and for b_2, he measures along direction $\hat{b}_2$ making an angle 315^0 with x-axis. For every set of question, they use one pair of singlet. They answer +1 if the result of spin measurement is up and -1 if it is down. Now on a singlet pair when Alice measure along direction $\hat{a}_i$ and Bob measures along $\hat{b}_j$, the probability that their results will be correlated (both results are up or both are down) is given by

$$P(up, up) = P(down, down) = \frac{1}{4}[1 - (\hat{a}_i.\hat{b}_j)]$$

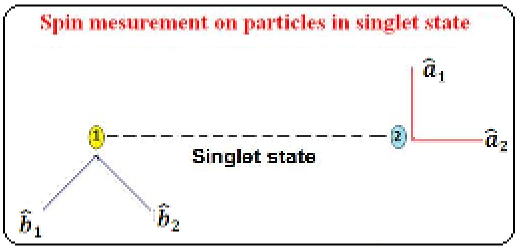

Fig.5

Then from quantum probability, one can easily calculate the success probability for winning the game.

$$P_{success}(a_1, b_1) = P_{success}(a_1, b_2) = P_{success}(a_2, b_1)$$
$$= P(up, up) + P(down, down)$$
$$= \frac{1}{4}[1 - Cos135^0] \times 2 = \frac{1}{2}(1 + \frac{1}{\sqrt{2}})$$
$$P_{success}(a_2, b_2) = P_{anti-correlation} = 1 - \frac{1}{4}[1 - Cos45^0] \times 2 = \frac{1}{2}(1 + \frac{1}{\sqrt{2}})$$

So probability of winning the game averaged over all questions with equal probability is given by

$$< P > = \frac{1}{4}[P_{success}(a_1, b_1) + P_{success}(a_1, b_2) + P_{success}(a_2, b_1) + P_{success}(a_2, b_2)]$$
$$= \frac{1}{2}\left(1 + \frac{1}{\sqrt{2}}\right) = 0.853$$

This shows that quantum correlation, though cannot help win this game deterministically, still perform something which is beyond the capacity of any local realistic theory.

It has been shown that the enforcing magic box correlation in quantum mechanics would imply violation of no-signaling condition [4]. Now we come to the question whether there is possibility of having the magic box correlation in some future physical theory. There is some recent result which shows that the existence of magic box would reduce all the communication complexity problem to triviality *i.e.* would be solved by one bit of communication [5]. From such results some people conclude that nature would not allow this correlation to exist.

4. Information Causality Principle

Recently a new causality principle namely information causality condition has been suggested by a group of scientists [6]. They have shown that all existing physical theories namely classical physics and quantum physics both obey this principle. Interestingly they have shown that some super quantum correlations like the magic box we discussed earlier, violate this principle. Let us see how this magic box which respects special relativity, still violate the information causality condition. First let us have a look at the information causality condition. Consider the following task. Alice is in a city where there are two cricket matches to be played, one between India and England and the other between Indian-junior and Srilanka. Bob is far away from the city but interested in learning the results of both the matches. Alice is to inform him the result at the end of the matches. How many bits (2 level systems) are required to convey the result of both the matches. One can easily check that two bits are necessary and also sufficient. A particular encoding is given in Figure 6.

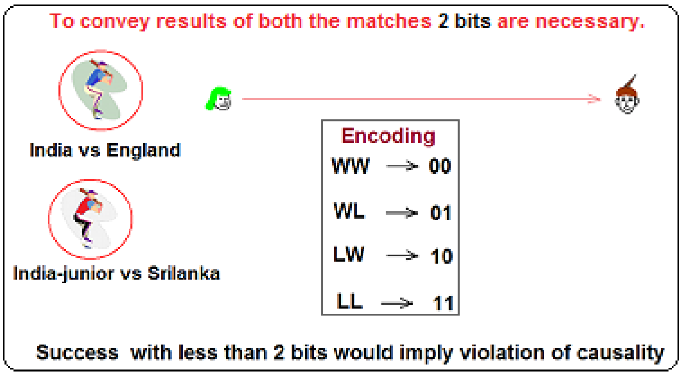

Fig.6

Now we design a different task. Alice watches both the matches. Bob will be asked to tell the result of one of the matches, decided randomly, after Alice communicates to Bob. Now the interesting question is the following; how many bits are required to be sent by Alice to Bob to enable him to tell the result of the match that he is asked, correctly? Two bits clearly suffice. Will one bit of communication suffice? This is the question, where information causality principle appears [6]. Information causality principle says that this task is impossible to perform. Still two bits of communication are required.

The above is the simplest extreme case for information causality principle. The actual statement made in the original article is the following: "The information gain that Bob can reach about a previously unknown to him data set of Alice, by using all his local resources and m classical bits communicated by Alice, is at most m bits." This statement can be expressed in the following way (Figure 8).

Alice receives N random bits $\hat{a} = (a_0, a_1, \dots \dots . a_{N-1})$. Each bit a_i $(i = 0,1..N-1)$ has value 0 or 1. In a separate location, Bob receives a random variable $n \in \{0,1,2 \dots \dots . N-1\}$. Alice is allowed to send m classical bits to Bob, with the help of which Bob has to guess the value of nth bit a_n chosen randomly from Alice's string of bits. Alice and Bob can share any correlated resources that respect no-signaling condition. Information causality limits the efficiency of solutions to this task. If $m < N$, then Bob's guess about a_r cannot be correct for all n. The actual limit to the success in this task put by information causality is far stronger and is expressed by bound on mutual information between the actual value of the bit and Bob's guess. The details in this regard is beyond the scope of the present essay.

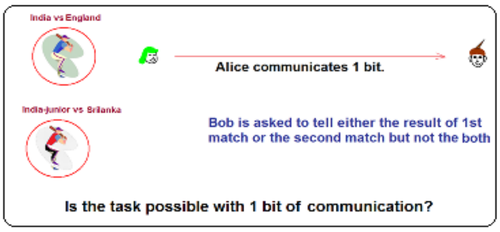

Fig.7

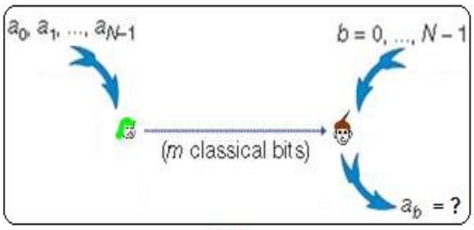

Fig.8

5. Protocol

It has been shown that in classical and also in quantum world this principle holds true. But at the same time, it has been shown that if Alice and Bob could share a pair of magic boxes, the job, suggested in Figure 7, would have been possible with one bit of communication. Let us see how (Figure 9). In case India wins or loses both the matches, Alice inputs a_1 to her part of the magic box. If India wins one of the matches, she inputs a_2. Alice communicates the result of the magic box directly to Bob if India wins in the first match. In case, India loses the first match, she flips the result of the magic box and communicates it to Bob. Obviously one bit is required for this communication. If Bob is asked the result of the first match, he inputs $b_1$and he inputs b_2 if he asked to tell the result of the second match. Finally, Bob will take the product of the result from his box and Alice's communicated result. The correspondence will be like this. If the product is +1, India wins and if it is -1, India loses. Now we shall check that this strategy really works for all possible cases.

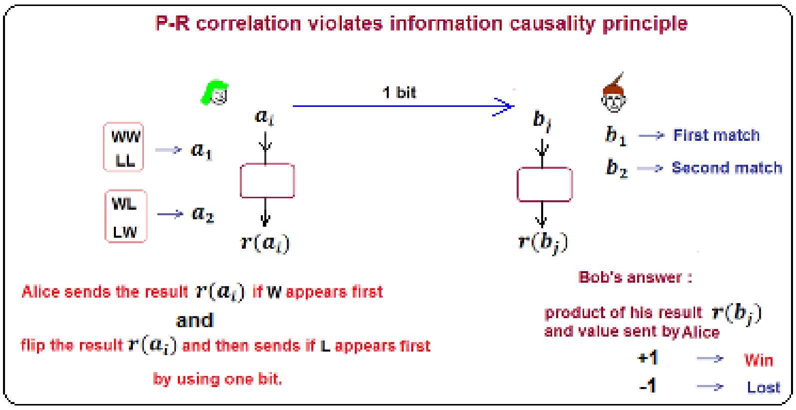

Fig.9

Case.1: Let India wins in both the matches. In this case Alice inputs a_1. If Bob is asked to tell the result of the first match, he inputs b_1 and he will input b_2 when he is asked the result of the second match. In both the cases Alice's result and Bob's result will be correlated, either both are +1 or both are -1 (Table 4). Alice will communicate her result to Bob and Bob will take the product of his result and the value sent by Alice which would obviously be +1 and Bob answers correctly.

Case.2: Let India loses in both the matches. In this case everything will be similar like the earlier cases excepting that Alice will communicate the flipped result ($\pm 1 \ \rightarrow \ \mp 1$) of his box. So, in both the cases, the product of Bob's result and the value sent by Alice will be -1 enabling Bob to answer correctly.

Case.3: Let India wins in the first and loses in the second match. In this case Alice inputs a_2 and sends the result directly. If Bob is asked to tell the result of the first match, he inputs b_1 and their result will be correlated and Bob will answer correctly. But if Bob is asked the result of the second match, he will input b_2 and in this case the result will be anti-correlated. So, Bob's product will be -1 enabling him to answer correctly.

Case.4: Let India loses in the first and wins in the second match. In this case Alice inputs a_2 and will send the flipped result. If Bob is asked to tell the result of the first match, he inputs b_1 and their results will be correlated and Bob's product will be -1 as Alice flips her result. But if Bob is asked the result of the second match, he will input b_2 and in this case the result will be anti-correlated. But Bob's product will be +1 again as Alice will send the flipped result. So, in both the cases, Bob will answer correctly.

6. Conclusion

Let us now summarize what we discussed. Quantum correlation, though respects no signaling condition, has the peculiar feature that it cannot be reproduced by any local realistic theory. Then people searched for all the possible correlations that respect no-signaling condition. It was found that the set of correlation that respect no-signaling condition is strictly larger than quantum correlation. The magic box correlation which we discussed in this article, is an example of such non-quantum correlation. Information causality principle suggests that this magic box correlation, though respects no-signaling condition as quantum mechanics does, cannot arise from a physical theory even in future. Actually, not only this magic box correlation, any super quantum correlation which helps to win the game with probability larger than that provided by quantum theory (85% approximately) can be dismissed by imposing the information causality principle [6].

References

1. A. Einstein, B Podolsky and N. Rosen, *Phys. Rev.*, **47** (1935) 777.
2. J. S. Bell, *Physics* (Long Island City, N.Y.), **1** (1964) 195.
3. S. Popescu and D. Rorlich, *Foundation of Physics*, **24**, (1994) 379-385.
4. Sujit K. Choudhary, Guruprasad Kar, Samir Kunkri & Ramij Rahaman *Phy. Lett.A.* **371** (2007) 395.
5. W. Van Dam, quant-ph/0501159.
6. M. Pawlowski, T. Paterek, D. Kaszlikowski, V. Scarani, A. Winter and M. Zukowski, *Nature*, **461** (2009) 1101–1104.